## THERMOPHYSICAL DESIGN OF THE PARAMETERS OF A COMPUTER BOARD WITH A MICROCHANNEL COOLING SYSTEM

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A procedure of thermophysical design and optimization of the parameters of a supercomputer board with a microchannel water system of convective cooling is developed.

To provide intense cooling of the most heat-stressed components of present-day computers, use of microchannel convective cooling systems made of high thermal conductivity materials is promising. Such systems of heat removal have shown great promise for cooling optical components of powerful technological lasers [1, 2] and find application for supercomputer boards [3-6]. Recently a number of different microchannel structures and their technologies have been developed. The microchannel cooling systems are characterized with channels sized to 0.1-1 mm, which in combination with the finning effect permits one to increase the heat transfer of boards up to  $(2-3) \cdot 10^5$  W/(m<sup>2</sup>·K) at a water filtration rate up to several meters per second. At such a heat transfer rate from a cooled wall heat fluxes up to  $10^7$  W/m<sup>2</sup> may be removed without heat-transfer agent boiling in microchannels.

A desire to obtain such high heat transfer rates leads to miniaturization of the cooling system, an increase of its manufacturing cost, and an elevated energy consumption for pumping of the cooling liquid. Such an approach is justified in the case of cooling powerful computer boards made of high-thermal conductivity structural materials when the total temperature difference does not exceed the temperature difference at the cooled wall-liquid interface. Below the reader is offered a thermophysical design of one possible construction of a cooled computer board. Thermal resistances of the board components are determined, thermal stresses developed in the construction material are evaluated, and the main parameters of the microchannel system of convective cooling with regard for the technology and assembly of boards are calculated.

Figure 1 shows one of the possible board constructions to be cooled. It consists of a heat-releasing crystal (1) and a rectangular BeO wafer (substrate) (2), to which the crystal is soldered. The substrate is in thermal contact with the tops of fins (3) all being built on an aluminum board (4). The space between the fins serves for a commutation network, whose influence on the heat conduction process in the fins may be neglected. The foundation of board (4) houses the microchannels of a convective cooling system (5) through which a heat-transfer agent is filtrated. The geometric and physical parameters of the construction under discussion are as follows:

Crystal dimensions, mm	$4 \times 4$
Substrate dimensions, mm	$13.5 \times 11$
Substrate thickness, mm	1
Thermal conductivity of the BeO substrate, $W/(m \cdot K)$	200
Contact layer thickness, mm	0.2
Thermal conductivity of the contact material, W/(m • K)	1
Fin cross section, mm	1.6 × 3
Fin height, mm	2
Thermal conductivity of the fins and Al board, $W/(m \cdot K)$	200
Thickness of the board wall to be cooled, mm	1
Dimensions of the region of the cooled board wall	
adjacent to the fin, mm	13.5 × 11

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Fig. 1. Schematic drawing of the supercomputer board with a microchannel convective cooling system.

Heat released in the crystal is transferred by conduction via the substrate, the fins and the foundation to the cooling system where it is taken away by the liquid. The maximum crystal temperature is determined by the temperature gradients across the construction elements and the contact thermal resistances at the boundaries of conjugation of the elements and the thermal head "cooled wall-liquid." To ensure a more correct determination of the maximum crystal temperature  $T_{max}$ , it is necessary to calculate the three-dimensional temperature distribution in the construction and to solve the equations of convective heat transfer in the cooling system. However this is extremely tedious.

To evaluate the crystal temperature  $T_{max}$ , we employ the approximate solutions based on the separate calculation of temperature fields in each construction element. In this case the total thermal resistance of the board is

$$R = \frac{T_{\rm c} - \bar{T}_l}{Q/S_{\rm c}} = R_{\rm c_1} + R_{\rm s} + R_{\rm c_2} + R_{\rm fnd} + R_{\alpha},\tag{1}$$

where  $R_{c_1}$  is the crystal-substrate contact thermal resistance;  $R_s$  is the thermal resistance to radial heat fluxes over the substrate;  $R_{c_2}$  is the contact thermal resistance at the substrate-fin top interface;  $R_{fnd}$  is the foundation thermal resistance (of the fins and the board);  $R_{\alpha}$  is the cooled wall-liquid thermal resistance (see Fig. 1).

The working temperature of the crystal must not exceed a value of  $[T] \approx 100^{\circ}$ C. Therefore the maximum permissible crystal power

$$Q = \frac{S_c}{R} \left( [T] - T_l \right) \tag{2}$$

is determined by the given permissible crystal temperature [T] and depends on thermal resistance R (1). The components of R (1) are determined below.

The lower the thermal resistance R (1) of the board is, the higher the power Q (2) that is removed from the crystal. Therefore on choosing the materials, geometric characteristics and contacts technologies of the board elements, one must tend to have minimal values of the components in the expression (1) for thermal resistance. The maximum permissible thermal power  $Q_{max}$  of the crystal (see (2)) for the construction depicted in Fig. 1 is attained at  $R_{min} = R_s + R_{fnd}$ , i.e., in the case of ideal thermal contacts at the boundaries of conjugation of the elements ( $R_{c_1} = R_{c_2} = 0$ ) and under intense heat release conditions ( $R_{\alpha} = 0$ ). Now we determine the components  $R_s$  and  $R_{fnd}$ 

Thermal resistance in the substrate  $R_s$  is most simply determined analytically by neglecting transverse temperature gradients as compared to longitudinal.  $R_s$  is determined by the relation

$$R_{\rm s} = \frac{S_{\rm c}}{4\pi\lambda_{\rm s}\delta_{\rm s}} \left[ 1 - \ln\frac{S_{\rm c}}{S_{\rm s}} \right]. \tag{3}$$

In order to decrease the thermal resistance of the substrate  $R_s$ , in accordance with (3) it is necessary: 1) to use materials with thermal conductivity  $\lambda_s$  as high as possible; 2) not to use thin substrates; 3) not to have close areas of the crystal and the substrate, i.e.,  $S_c/S_s \approx 1$ .



Fig. 2. Influence of the relative area of the heat-releasing crystal  $S_c/S_s$  on the relative increase of the substrate thermal resistance. Calculation by formula (4).

The latter conclusion is illustrated by Fig. 2, which shows that for the relative area  $0.1 < S_c/S_s < 0.6$  the substrate thermal resistance is 3.5-1.5-fold higher than the minimal value  $R_s(1) = S_c/4\pi\lambda_s\delta_s$  when the dimensions of the crystal and the substrate are equal.

For a crystal mounted on the BeO substrate (see the conclusions above), the thermal resistance of the substrate is  $2 \cdot 10^{-5} \text{ K} \cdot \text{m}^2/\text{W}$  which at a thermal capacity of 10 W corresponds to a temperature drop of  $12^{\circ}\text{C}$ .

Thermal resistance of the foundation may be evaluated neglecting two-dimensional effects in the formation of a temperature field in the fin and the board by the following expression:

$$R_{\rm fnd} = \frac{S_{\rm c}}{S_{\rm f}} \frac{\delta_{\rm f}}{\lambda_{\rm f}} + R_{\rm b} , \qquad (4)$$

where

$$\begin{split} R_{\rm b} &= \frac{1}{\alpha_{\rm ef}} \left[ S_{\rm c} \left( \frac{1}{S_{\rm f}} - \frac{1}{S_{\rm b}} \right) - \frac{2C_0 I_1(X_1)}{X_1} \right] \quad \text{at} \quad \mathrm{Bi} < 1 \\ C_0 &= \frac{K_1(X_1)/I_1(X_1) - K_1(X_2)/I_1(X_2)}{K_0(X_1) + K_1(X_1) I_0(X_1)/I_1(X_1)}, \\ X_1 &= \sqrt{\frac{\alpha_{\rm ef} S_{\rm f}}{\pi \lambda_{\rm b} \, \delta_{\rm b}}}, \quad X_2 = \sqrt{\frac{\alpha_{\rm ef} S_{\rm b}}{\pi \lambda_{\rm b} \, \delta_{\rm b}}}. \end{split}$$

In (4), Bi =  $\alpha_{ef}\delta_b/\lambda_b$  is the Biot number for the board.

The first term in (4) is the thermal resistance of the fin, while the second term is the thermal resistance due to heat fluxes over the cooled wall of the foundation. The expression for evaluation of  $R_b$  is the result of the solution of the onedimensional heat conduction equation in cylindrical geometry in the approximation of small Biot numbers. It is obvious that at  $S_f = S_b$  the thermal resistance is  $R_b = 0$  since in this case there are no radial heat fluxes in the wall.

For the aluminum foundation thermal resistance (see the conclusions above)  $R_{fnd}$  is 3.4  $\times$  10<sup>-5</sup> K·m<sup>2</sup>/W.

The maximum permissible thermal capacity of the crystal Q as a function of the crystal temperature [T] is evaluated by expressions (2)-(4)

[T] — T <sub>l</sub> , ℃	<b>20</b> °	50	70	100
<b>Q</b> , W	6	15	,21	30



Fig. 3. Ratio of the pressure drop over the cooled layer  $\Delta P_0$  to that along the collector  $\Delta P_c$  for the 110  $\times$  90  $\times$  4 mm board versus the hydraulic diameter of the microchannels d<sub>h</sub> at a constant water flow rate of 50 g/sec. The cross section of the waffle structure fin is 1  $\times$  1 mm, of the collectors 2.5  $\times$  4 mm.  $\delta_h$ , mm.

Fig. 4. Permissible thermal capacity Q of the crystal versus the contact thermal resistance  $\delta_{c1}/\lambda_{c1}$  at different crystal temperatures [T]. Solid lines, for a crystal with dimensions  $S_c = 4 \times 4$  mm, dashed line,  $S_c = 9 \times 10$  mm. Q, W;  $\delta/\lambda$ , m<sup>2</sup> · K/W.

The values of Q represent the physical sense of the thermal factor for the cooled board construction depicted in Fig. 1. In practice permissible Q values may be much lower because of substantial thermal resistances developed between the board elements and the insufficient efficiency of the cooling system in the case of the nonoptimal choice of its parameters.

In order to remove integral heat loads of several hundred of watts and to maintain a reasonable crystal temperature, it is advantageous to use microchannel convective cooling systems [1-7]. In Fig. 1 microchannel system (5) is shown convectionally as long rectangular channels. At present a large group of different microchannel structures exists which provides the effective heat transfer up to  $10^5 \text{ W/(m^2 \cdot K)}$  when water is cooled at a filtration velocity up to several meters per second [1, 2].

On choosing the structure, geometric sizes of channels, a construction of liquid supply and its consumption for the board depicted in Fig. 1, one must observe the following requirements:

1. The absence of stagnant zones with deteriorated heat transfer in the cooled layer, which is necessary for the implementation of uniform liquid distribution in microchannels.

2. Cooled wall-liquid thermal resistance  $R_{\alpha}$  must not exceed the minimal thermal resistance of the board  $R_{\min} = R_s + R_{fnd}$ , i.e.,  $R_{\alpha} < R_{\min}$ .

3. Liquid heating temperature  $T_l$  must not exceed the temperature difference in the board elements, i.e.,  $\Delta T_l < qR_{min}$ .

4. The pressure drop for a liquid flowing via the microchannels of the cooled layer and the collector channels is limited by a value of several tenths of an atmosphere.

The last two requirements are caused by the need to ensure the permissible crystal temperature and the mechanical strength of the construction of terminal plates when implementing the series connection of cooling systems of several boards into a self-contained cooled circuit.

The most promising finned structure for the cooled layer of the board is that of a waffle type [1]. Such a structure is formed when rectangular channels on the rear side of the board base are milled in two mutually perpendicular directions. As a result, a system of rectangular tenons is formed with a cross liquid flow around them. After milling, the finned surface is closed with a plate. Ideally, the assembly technology must provide the mechanical and thermal contact of the fins tops of the waffle structure and the plate.

From the viewpoint of heat transfer enhancement, finning in the form of "a waffle" has a number of advantages over other structures, including: 1) high coefficients  $\alpha$  of heat transfer from a surface of rectangular fins with a cross liquid flow around them; 2) high cross heat conductivity of the carcass of tenons of the porous layer; 3) vigorous mixing of the heat-transfer agent and, as a result, enhanced heat transfer from walls in the interfin space.

TABLE 1. StationaryValues of theTemperature Field

Points in Fig. 1	t, °C	σ, MPa
A	51	+5
В	47	14
С	24	1
D	22,5	51
Ε	22	36

The above factors make it possible to obtain high effective heat transfer coefficients  $\alpha_{ef}$  over waffle structures. The cooled wall-liquid thermal resistance  $R_{\alpha}$  for a waffle structure may be evaluated by the following relation:

$$R_{\alpha} = \frac{1}{\alpha_{\rm ef}} = \frac{1}{\alpha} \left[ \frac{(d_{\rm c} + d_{\rm f})^2}{2 (d_{\rm c} (d_{\rm c} + 2d_{\rm f}) + 2h_{\rm f} d_{\rm f})} \right].$$
(5)

The expression in square brackets specifies the degree of cooled surface development at the expense of finning and is obtained provided there is weak temperature attenuation with respect to the height of the cooled fin.

The coefficient of heat transfer  $\alpha$  from the surface of fins may be determined by the relation [8]

$$\alpha = 1.55 \left[ \frac{-\text{Pe}\,d_{\text{h}}}{d_{\text{f}}} \right]^{1/3} \frac{\lambda}{d_{\text{h}}},\tag{6}$$

where  $Pe = vd_h/a$  is the Peclet number. This expression is obtained for the mean heat transfer coefficient on the starting length of the channel at a constant boundary condition of the first kind, i.e., at  $(d_f/d_hPe) \le 0.05$ .

For the waffle aluminum structure with a width of microchannels of  $d_c = 0.5$  mm, a fin thickness of  $d_f = 1$  mm, and a fin height of  $h_f = 0.5$  mm cooled with water at a flow velocity of v = 2.5 m/sec the heat transfer coefficient  $\alpha$  is 30000 W(m<sup>2</sup>·K), while thermal resistance  $R_{\alpha}$  is  $1.6 \times 10^{-5}$  m<sup>2</sup>·K/W. This value is lower than the minimal thermal resistance of the board  $R_{min}$ , i.e.,  $R_{\alpha} < R_{min}$ .

Uniformity of liquid distribution in the microchannels of a cooled layer may be attained by the correct choice of the liquid supply scheme and of the geometric sizes of microchannels and collectors. The constancy of velocity v(z) in each section of a cooled layer x is achieved only in the case when the pressure drop in the collectors  $\Delta P_c$  is much less than that in the cooled layer  $\Delta P_0$  ( $\Delta P_c < < \Delta P_0$ ). At comparable pressure drops  $\Delta P_0$  and  $\Delta P_c$  that take place in practice, almost uniform liquid consumption over the section x is possible only for the Z-shaped scheme of liquid supply. In this case the inlet and outlet of a heat-transfer agent are arranged on the opposite sides of the board.

Figure 3 shows the dependence of the ratio of the pressure drop on the cooled layer  $\Delta P_0$  to that along the collector  $\Delta P_c$  for the 110 × 90 × 4 mm board on the hydraulic diameter of the cooled layer microchannels at a constant fin thickness of waffle structure  $d_f = 1$  mm. The collectors are positioned along the short side of the board; dimensions of their cross section are 2.5 × 4 mm. It is seen that already at microchannel diameters smaller than 0.6 mm,  $\Delta P_0$  is essentially larger than  $\Delta P_c$ , i.e., the choice of a diameter of the cooled layer microchannels of less than 0.6 mm provides uniform liquid distribution in the microchannels. On the other hand, when a waffle structure with a microchannel diameter  $d_h$  of less than 0.5 mm is used, the hydraulic resistance of the cooled layer impermissibly increases (up to several atmospheres at a water flow rate of 50 g/sec via the board).

Thus, the cross section of the collectors and the size of the microchannels of the cooled layer are rigidly bound. The size of a collector is mainly specified by the thickness of the board base (4 mm), while its height and width cannot exceed 2-4 mm. Therefore if the cross section of the collector is  $2.5 \times 4$  mm and the fins of the waffle structure have a thickness d<sub>f</sub> = 1 mm, the optimum size of the microchannels is  $0.5 \times 0.5$  mm.

If water flows at a rate of 50 g/sec through a board cooling system with  $0.5 \times 0.5$  mm microchannels (d<sub>f</sub> = 1 mm) and  $2.5 \times 4$  mm collectors, the pressure drop  $\Delta P = 2\Delta P_c + \Delta P_c$  is  $0.37 \cdot 10^5$  Pa, and if the total heat release in the board crystals is 500 W, the liquid does not heat up more than  $2.5^{\circ}$ C.

The board structure shown in Fig. 1 provides for contact with elements in two locations: at the crystal-substrate and substrate-fin tip boundaries. Contact thermal resistances  $R_{c1}$  and  $R_{c2}$  are created in the contact zone and can be estimated from the expressions

$$R_{c_1} = \delta_{c_1} / \lambda_{c_1}$$
 and  $R_{c_2} = \frac{4S_f}{S_c} \frac{\delta_{c_2}}{\lambda_{c_2}}$ ,

where  $\delta_{c_{1,2}}$  and  $\lambda_{c_{1,2}}$  are the thickness and thermal conductivity of a solder or an adhesive providing the mechanical and thermal contacts. If the contacts are provided by high thermal conductivity solders (e.g., silver) and the soldering technology allows sound joints to be provided over the entire contact surface (without air space), then the contact thermal resistances  $R_{c1}$ and  $R_{c2}$  are essentially less than that of the board and therefore they may be neglected. Figure 4 shows the permissible thermal capacity of the crystal Q (2) (at  $R = R_s + R_{c2} + R_f$ ) versus the thickness and heat conduction of the substrate-fin base contact. The l.h.s. of this plot  $[\delta_{c2}/\lambda_{c2} (10^{-6}-10^{-5}) (m^2 \cdot K)/W]$  corresponds to the case of good contacts. Here Q weakly depends on  $\delta_c/\lambda_c$ . If the contact is provided by a low thermal conductivity adhesive (e.g. epoxy resin), then the contact resistances  $R_{c1}$  and  $R_{c2}$  may essentially exceed the board thermal resistance  $R_{min}$ . In this case the permissible thermal capacity of the crystal mainly depends on the thermal resistance of the contacts (see the r.h.s. of Fig. 4 at  $\delta_{c2}/\lambda_{c2} > 2 \cdot 10^{-5}$  $m^2 \cdot K/W$ ).

For the contact formed by the 0.2 mm thick epoxy resin layer with a filler  $[\lambda_{c2} = 1 \text{ W/(m \cdot K)}]$  the permissible thermal capacity Q (2) of the 4 × 4 mm crystal is 2.5-5 W, dependent on a permissible temperature of 50-90°C. This is almost a factor of 8 lower than the permissible crystal power Q in the case of ideal contacts in the board construction.

In the board construction, materials with different mechanical properties are used and the considerable contact resistances cause an increase of the temperature of the construction elements up to several tens of degrees. Therefore impermissible thermoelastic stresses may develop in the board due to the temperature gradients in the construction and the difference in mechanical properties of the materials: thermal expansion coefficients  $\beta$ , 1/K and elasticity moduli E, N/m<sup>2</sup>.

The design of the thermostressed state of the construction faces two main problems, namely, the complexity of the geometry and the presence of a thin adhesive layer. For this, sufficiently exact stress estimates may be obtained by replacing a real construction by the two-dimensional one in which a two-dimensional-stressed state is realized. After such simplification, a complex of two-dimensional finite-element programs may be used [9] which includes the calculation of nonsteady-state distributions of the temperature and the thermal stresses. One of the merits of this packet of programs is the possibility of subdivision of the construction under study into a large number of elements ( $\sim 1000$ ), which allows investigation of the contribution of thin layers (welding, soldering, adhesive sites) to the thermostressed state.

Calculations have shown that practically all the temperature difference in the board takes place mainly over a thin ( $\delta = 0.2 \text{ mm}$ ) epoxy resin layer. Thus, for the tested materials at characteristic values of the heat flux ( $q = 10^5 \text{ W/m}^2$ ) and the coefficient of heat transfer from a substrate ( $\alpha = 10^4 \text{ W/(m^2 \cdot K)}$ ) this temperature difference  $\Delta T \cong 24^{\circ}\text{C}$ .

The stationary values of the temperature field at the points designated in Fig. 1 as well as the values of thermoelastic stresses occurring in the board at the given temperature distribution are listed in Table 1.

We may draw the conclusion that for the conditions considered the stresses close to the maximum permissible values are already attained in the construction. Besides, the tensile stresses at the point A near the crystal ( $\sigma \approx 5$  MPa) and the compressive stresses ( $\sigma \approx -14$  MPa) in the epoxy resin (point B) are thought of as being the most dangerous.

The stresses at these points and the temperature level may be considered to be the main factors restricting an increase of heat fluxes for the board construction under consideration.

As has been mentioned, the stressed state is determined not only by the temperature field but also by the mechanical properties of the joined materials. In particular, considerable tensile stresses in the epoxy resin are caused by its high thermal expansion coefficient  $\beta$  as compared to the expansion coefficients of BeO and Al contacting with it. Therefore, to connect BeO and Al, material with a lower thermal expansion coefficient should be used. This allows the level of stresses (with respect to a module) to be decreased without, however, exerting essential influence on the other restricting parameters, namely, the stress near the crystal and the temperature, whose high level is determined by low thermal conductivity typical for the majority of known adhesives.

Thus, the procedure of the thermophysical design of the supercomputer board with a microchannel system of convective cooling is developed. The analytical relation is established between the geometrical and physical parameters of the board, its power and temperatures, heat-transfer agent consumption, and the hydraulic resistance of the cooling system. The limiting thermal capacity of the crystal for the board construction under consideration is evaluated to be 20 W (15 W) at a crystal temperature of 90°C (70°C). Also, the advantages of the microchannel convective cooling system with a waffle-type aluminum structure with the  $0.5 \times 0.5$  mm ( $1 \times 1$  mm) channels (fins) and the  $2.5 \times 4$  mm collectors are given. At a water flow rate of 50 g/sec the cooling system provides the effective heat transfer of about  $0.6 \times 10^5$  W/(m<sup>2</sup>·K) with a uniform liquid consumption over the entire board surface and water heating up to  $2.5^{\circ}$ C (Q = 500 W).

Contact thermal resistances at the crystal substrate-fins tops interface exert no appreciable influence on the permissi-

ble thermal capacity of the crystal only at the thermal contact resistance  $\delta_c/\lambda_c \leq (10^{-6}-10^{-5}) \text{ m}^2 \cdot \text{K/W}$ . If  $\delta_c/\lambda_c > 2 \cdot 10^{-5} \text{ m}^2 \cdot \text{K/W}$ , the permissible thermal capacity is smaller by 3-4 times and more.

In the contact material (epoxy resin) at a thermal capacity of 1-2 W of the crystal, thermal stresses develop ( $\sim 15$  MPa) which are close to the limiting permissible values. As a consequence, the contact may fail due to cyclic heat loads over the crystal.

## NOTATION

 $S_s$ , substrate area;  $\delta_s$ , its thickness;  $\lambda_s$ , substrate thermal conductivity;  $S_c$ , crystal area;  $T_c$ , temperature at the crystal center;  $\overline{T}_l$ , mean liquid temperature; Q, thermal capacity of the crystal;  $\alpha_{ef}$ , effective heat transfer coefficient of the board;  $S_f$ ,  $S_b$ , cross-sectional area of the fin and the board wall adjacent to the fin, respectively (Fig. 1);  $\delta_f$ , fin height;  $\delta_b$ , upper wall thickness of the board;  $\lambda_f$ ,  $\lambda_b$ , thermal conductivity of the fin and the board, respectively;  $I_0$ ,  $I_1$ ,  $K_0$ ,  $K_1$ , modified Bessel functions of the first and second kind;  $d_c$ , microchannel width;  $d_f$ ,  $h_f$ , fin thickness and height, respectively; v, liquid velocity in a microchannel; a,  $\lambda$ , liquid thermal diffusivity and thermal conductivity, respectively;  $d_h = 4F/\Pi$ , hydraulic diameter of a microchannel; F and  $\Pi$ , area and perimeter of its cross-section.

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